

## Hybrid method for solving incomplete linear optimization problem on distributed memory multiprocessor system\*

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### Abstract

*An approach to solving the linear optimization problem with unformalized restrictions is proposed. This approach is oriented to distributed memory multiprocessor systems. A parallel algorithm, based on combination of linear programming and discriminant analysis methods, is described. The results of computing experiments confirming the efficiency of our approach are presented.*

### 1. Introduction

In the sphere of economic-mathematical simulations, we often have to solve some problems, which demand to take into account a big number of correlated factors including some unformalized restrictions [2]. The process of solving an unformalized problem supposes converting a formulation of the problem into a more formal view by using some simplifications and adjustments. As a result, we get a formalized problem, which is approximating primary unformalized problem.

In many cases, we can construct the formalized problem in a form of linear programming problem. However, the solution point of the formalized problem can be arbitrary far from the solution point of the primary problem due to existence of unformalized restrictions.

In such situation, we can use a hybrid method based on combination of linear optimization methods and expert estimation procedures. The required algorithm is built as an iterative process, which refines the unformalized restrictions step by step. Intermediate solving points obtained during the iterative process form a set of patterns, which were qualified by an expert. Applying a discriminant analysis procedure [4] to this set of patterns, we get a separating hyperplane approximating one unformalized restriction. Therefore, the important subject is to design a new hybrid algorithm for solving linear programming problems with unformalized restrictions by force of combining linear optimization and discriminant analysis methods.

In papers [5, 3], a general method of solving problem with one unformalized restriction was described. We proposed an iterative implementation of the general method called SM-LC algorithm. It's exploiting the combination of simplex-method and linear correction method.

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In this paper, theoretical and implementation aspects of parallel edition of SM-LC algorithm are being discussed.

### 2. Mathematical model

Let the model of economic circumstances be described by a linear optimization problem for that we couldn't completely formalize several restrictions. We can represent such model as the following Linear Programming problem with Unformalized Restrictions (LPUR) in  $\mathbf{R}^n$  space.

$$\bar{x} \in \text{Arg max} \left\{ (c, x) \mid Ax \leq b; \bar{\phi}_i(x) \leq 0; i = 1, \dots, q \right\}. \quad (1)$$

Here, inequalities  $\bar{\phi}_i(x) \leq 0$  ( $i = 1, \dots, q$ ) act a role of unformalized restrictions in such a way where we don't now views of the functions  $\bar{\phi}_i$ . Let us suppose functions  $\bar{\phi}_i$  belonging to affine function class. We also suppose that problem (1) has unique solution.

Let  $D = \{x \mid Ax \leq b\}$  be a polyhedron defined by the formalized part of the problem LPUR, let  $D$  be a bodily set (i.e.  $D$  has internal points) and let  $D$  be a bounded set.

We will designate the problem

$$\max \{ (c, x) \mid Ax \leq b \}. \quad (2)$$

as *Linear Programming problem with Formalized part (LPF)*.

We will suppose problem (2) having the unique solution  $\hat{x} = \arg \max \{ (c, x) \mid Ax \leq b \}$  and  $\hat{x} \neq \bar{x}$ . Let

$\bar{P}_i = \{x \mid \bar{\phi}_i(x) \leq 0\}$  be a set of points satisfying unformalized restriction  $\bar{\phi}_i(x) \leq 0$ . Since  $\bar{\phi}_i$  is an affine function,  $\bar{P}_i$  is a closed half-space. Let

$$\bar{P}^* = \bigcap_{i=1}^q \bar{P}_i.$$

Then, the admissible set of LPUR problem is  $D \cap \bar{P}^*$ . We suppose  $D \cap \bar{P}^*$  having internal points. The following stability theorem for the case of multiple unformalized restrictions is valid.

**Theorem 1.** LPUR problem (1) is stable on  $\bar{\phi}_1, \dots, \bar{\phi}_q$ .

*Proof* is similar to the proof of the theorem for the case of single unformalized restriction [3].

Let us demonstrate how solving LPUR problem can be reduced to solving a set of problems with single unformalized restriction.

Let us concede the following set of *independent* LPUR problems with a single unformalized restriction:

$$LPUR_1 : \bar{x}_1 = \arg \max \{(c, x) | Ax \leq b, \bar{\phi}_1(x) \leq 0\};$$

$$\dots$$

$$LPUR_q : \bar{x}_q = \arg \max \{(c, x) | Ax \leq b, \bar{\phi}_q(x) \leq 0\},$$

(we suppose that each of these problems has an unique solution). Using  $\mathfrak{L}$  algorithm for patterns generation [3], we can build a sequence of affine *separating* functions  $\tilde{\phi}_1^1, \dots, \tilde{\phi}_i^k$  to every LPUR<sub>*i*</sub> problem ( $i = 1, \dots, q$ ).

Let us construct the following linear programming problem  $LP^k$ :

$$\tilde{x}^k \in \text{Arg} \max \{(c, x) | Ax \leq b, \tilde{\phi}_i^k \leq 0; i = 1, \dots, q\}. \quad (3)$$

The following theorem is valid.

**Theorem 2.** Let LPUR problem (1) have the unique solution  $\bar{x}$ . Let every problem LPUR<sub>*i*</sub> ( $1 \leq i \leq q$ ) have the unique solution  $\bar{x}_i$  and  $\bar{x}_i \neq \hat{x}$ , where  $\hat{x} = \arg \max \{(c, x) | Ax \leq b\}$  is unique solution of LPF problem. Let  $\tilde{x}^k$  be a solution of  $LP^k$ . Then

$$\{\tilde{x}^k\}_{k=0}^{\infty} \rightarrow \bar{x},$$

i.e. if  $k$  is large enough then the solution of  $LP^k$  (3) can be used as an approximate solution of LPUR problem (1).

In formulation of theorem 2, we supposed *all* the problems LPUR<sub>*i*</sub> having a solution  $\bar{x}_i$  not equal to  $\hat{x}$ , which is the solution of LPF problem (2). In practice, we can realize this demand in the following way.

At the first step, we calculate approximate values of the coefficients and absolute term for all the unformalized restrictions  $\bar{\phi}_i(x) \leq 0$  satisfying condition  $\bar{x}_i \neq \hat{x}$  (each restriction is calculated *independently* from others). Let us notice that if no one unformalized restriction satisfies condition  $\bar{x}_i \neq \hat{x}$  than  $\hat{x}$  is the solution point of LPUR problem.

At the second step, the *approximations* of unformalized restriction received at the first step have to be added to system of restrictions of LPF problem.

Now let  $\hat{x}$  be the solution of extended LPF problem. Among unused unformalized restrictions, we choose those, for which condition  $\bar{x}_i \neq \hat{x}$  is true, and calculate approximations for them.

The process is finished when all unformalized restrictions have to be found. The final extended LPF problem gives us the approximate solution point for the primary LPUR problem.

### 3. Parallel edition of SM-LC algorithm

Let us describe a parallel algorithm of solving LPUR problem. At the first step, LPUR problem with  $q$  unfor-

malized restrictions is divided into  $q$  problems: LPUR<sub>1</sub>, ..., LPUR<sub>*q*</sub>. Each of these problems is independently solved by SM-LC method. Using calculated approximations of the unformalized restrictions and the formalized part of LPUR problem, we build the complete system of restrictions. At the second step, the problem obtained in such way is solved by simplex method. As a result, we have got an approximation of solving point  $\bar{x}$  for primary LPUR problem.

Calculating of each unformalized restriction was implemented as an independent process. The calculated restrictions were sent to a specified coordinator processor node, which built and solved the complete linear programming problem by simplex method. Such approach allows us to execute the program on arbitrary count of processors without any modifications. To manage the interprocessor communications, we used MPI-2 package. Described parallel SM-LC algorithm can be executed on shared memory multiprocessor system as well as on distributed memory multiprocessors including clusters.

### 4. Experimental results

For the computational experiments, the following model problem  $Mod-n_q$  was constructed:

$$\begin{cases} x_1 - 2x_2 \leq 0 \\ \vdots \\ x_1 - 2x_n \leq 0 \\ x_1 + 2x_2 \leq 8000 \\ \vdots \\ x_1 + 2x_n \leq 8000 \\ x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0 \\ Q_{\max} = x_1 \\ \begin{cases} e_1(x) = \text{sgn}(3x_1 - x_2 - 3000) \\ \vdots \\ e_q(x) = \text{sgn}(3x_1 - x_q - 3000) \end{cases} \end{cases}$$

Here,  $e_1, \dots, e_q$  are expert function defining the unformalized restrictions ( $q \leq n - 1$ ).  $Mod-n_q$  is a LPUR problem in space  $\mathbf{R}^n$  having  $q$  unformalized restrictions.

For our experiments, we used computational cluster Infinity [1]. We investigated the acceleration and scalability of parallel SM-LC algorithm exploiting the combination of simplex method and linear correction method. The results of acceleration evaluation are presented on Figure 1. We investigate three dimensions:  $n = 60$ ,  $n = 80$  и  $n = 100$ . The number of unformalized restrictions in all cases was  $q = 48$ .

The acceleration was calculated by formula  $\alpha = \alpha_0 + t_p / t_8$ , where  $t_8$  is the time of solving  $Mod-n_{48}$  problem on 8 processors,  $t_p$  is the time of solving the

same problem on  $p$  processors,  $\alpha_0 = (n-60)/60$ . In all cases we got the acceleration near to linear one. The results of scalability evaluation are presented on Figure 2. We investigate the same three dimensions:  $n=60$ ,  $n=80$  и  $n=100$ . Here  $t_p$  denotes the time of solving model problem with  $q=p$  unformalized restrictions on  $p$  processors.

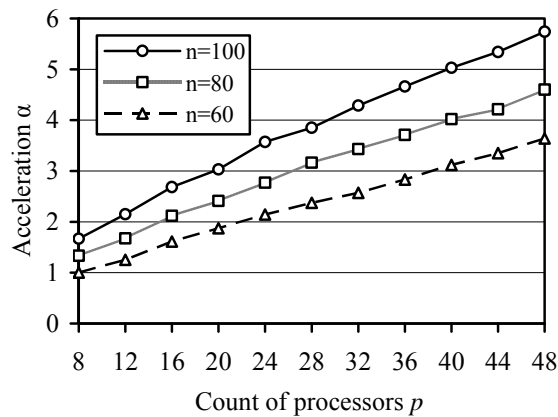


Figure 1. Acceleration for Mod- $n_{48}$ .

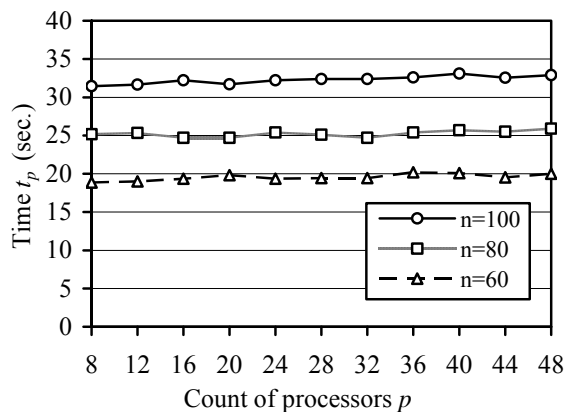


Figure 2. Scalability for Mod- $n_p$ .

The experiments showed that parallel SM-LC algorithm demonstrates the scalability near linear one in all cases.

## 5. Conclusion

An iterative SM-LC algorithm was proposed. It can be used for solving linear programming problem when part of restrictions can't be formalized. SM-LC algorithm is based on combining simplex method and linear correction method. SM-LC algorithm was initially designed for the case of single unformalized restriction. In this paper, we described a generalization of the algorithm for the case of multiple unformalized restrictions. The parallel edition of

SM-LC algorithm was presented. It can be used on distributed memory multiprocessor as well as on shared memory multiprocessor. The computational experiments on cluster multiprocessor system were done. The experimental results approved the effectiveness of the proposed approach.

## 6. References

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